647510 **MSC2MatC201x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
May/June -2021 [NEW COURSE]
Algebra – 2 (CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (A) Answer the following question. **[04]**

(1) Define normal extension and separable extension of a field.

Q.1 (B) Answer any two of the following questions. **[10]**

(1) Define algebraic extension of a field and prove that if L is an algebraic extension of K and K is an algebraic extension of F then L is an algebraic extension of F.

(2) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

(3) Define splitting field and find degree of splitting field of the polynomial $x^4 - 2$ over Q.

Q.2 (A) Answer the following question. **[04]**

(1) Find order of the group $G(Q(2^{\frac{1}{3}})/Q)$.

Q.2 (B) Answer any two of the following questions. **[10]**

(1) Let $f(x) \in F[x]$ has r distinct roots in its splitting field E over F. Then prove that the Galois group $G(E/F)$ of $f(x)$ is a subgroup of the symmetric group S_r.

(2) Show that the Galois group of the polynomial $x^4 + 1 \in Q[x]$ is the Klein four group.

(3) State and prove fundamental theorem of algebra.

Q.3 (A) Answer the following question. **[04]**

(1) Show that every additive abelian group is a Z module.

Q.3 (B) Answer any two of the following questions. **[10]**

(1) Define direct product of modules and show that direct product of two R-modules is an R - module.

(2) Let M be an R- module and $x \in M$. Construct smallest R -submodule of M containing x.

(3) Let $\{ N_i\}_{i\in\Lambda}$ be a family of R submodules of an R- module M. Show that $\bigcap_{i\in\Lambda} N_i$ is also an R submodule.

Q.4 (A) Answer the following question. **[04]**

(1) Show that the set of generators of a module may not be unique.

Q.4 (B) Answer any two of the following questions. **[10]**

(1) Define quotient module and show that the quotient map is a module homomorphism.

(2) State and prove fundamental theorem of R- homomorphism.

(3) Let R be a ring with unity and M be an R-module. Then prove that M is simple if and only if $M \cong R/I$ Where I is a maximal left ideal of R.

Q.5 (A) Answer the following question. **[04]**

(1) State fundamental theorem of Galois theory.

Q.5 (B) Answer any two of the following questions. **[10]**

(1) Define a finitely generated module and give an example of it.

(2) Show that the group $G(Q(\alpha)/Q)$ Where α is a nonreal fifth root of unity is isomorphic to cyclic group of order 4.

(3) Let G be a cyclic group. Show that G (as a Z module) has a basis if and only if G is finite.

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